Confining strings in the Abelian-projected $SU(3)$-gluodynamics II. 4D-case with θ-term

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Abstract. – The generalization of 4D confining string theory to the $SU(3)$-inspired case is derived. It describes string representation of the Wilson loop in the $SU(3)$-analogue of compact QED extended by the θ-term. It is shown that although the obtained theory of confining strings differs from that of compact QED, their low-energy limits have the same functional form. This fact leads to the appearance of the string θ-term in the low-energy limit of the $SU(3)$-inspired confining string theory. In particular, it is shown that in the extreme strong coupling regime, the crumpling of string world sheets could disappear owing to the string θ-term at $\theta = \pi/12$. Finally, some characteristic features of the $SU(N)$-case are pointed out.

One of the ways to construct string representation of QCD is to use the method of Abelian projections [1] (see ref. [2] for recent reviews). The assumption of monopole condensation employed within this method has recently been proved with a high accuracy by the lattice calculations in ref. [3]. To model the condensation of Abelian-projected monopoles one can treat them as a grand canonical ensemble with the Coulomb interaction. In the 3D $SU(3)$-inspired case [4], this led to a certain generalization of the theory of confining strings [5], which was originally invented for the description of confinement of electric charges in 3D compact QED [6]. The aim of the present letter is to extend the results of ref. [5] to the realistic 4D case with the inclusion of θ-term. The model in which confining strings will be studied is thus nothing else, but the $SU(3)$-version of 4D compact QED with θ-term. In this way, we shall see that similarly to the case of compact QED [7], in the $SU(3)$-inspired model the field-theoretical θ-term generates eventually string θ-term. At a certain critical value of θ, the latter one might be important for getting rid of crumpling of the string world sheet.

Consider first the case without the θ-term. Within the so-called Abelian dominance hypothesis [8], the $SU(3)$-inspired theory under study describes free diagonal gluons and Abelian-projected monopoles (*cf.* the 3D-case [4, 9]). The action of this theory can be written as follows ([1]):

([1]) Throughout the present letter, all the investigations will be performed in the Euclidean space-time.

$$\frac{1}{4}\int d^4x\mathbf{F}_{\mu\nu}^2 + \frac{1}{2}\int d^4x\int d^4y\mathbf{j}_\mu^{\mathrm{mon}}(x)D_0(x-y)\mathbf{j}_\mu^{\mathrm{mon}}(y). \tag{1}$$

Here, $\mathbf{F}_{\mu\nu} = \partial_\mu\mathbf{A}_\nu - \partial_\nu\mathbf{A}_\mu$ is the field strength tensor of diagonal gluons $\mathbf{A}_\mu = \left(A_\mu^3, A_\mu^8\right)$ and $D_0(x) = 1/(4\pi^2x^2)$ is the 4D massless propagator. Besides that, in eq. (1) the N-monopole current $\mathbf{j}_\mu^{\mathrm{mon}}(x)$ is defined for $N > 0$ as follows:

$$\mathbf{j}_\mu^{\mathrm{mon}}(x) = g_m\sum_{a=1}^{N}\mathbf{q}_{\alpha_a}\oint dz_\mu^a(\tau)\delta\left(x - x^a(\tau)\right), \tag{2}$$

whereas for $N = 0$, $\mathbf{j}_\mu^{\mathrm{mon}}(x) \equiv 0$. Here, the magnetic coupling constant g_m is related to the QCD coupling constant g by the equation $gg_m = 4\pi$. Next, in eq. (2), we have parametrized the trajectory of the a-th monopole by the vector $x_\mu^a(\tau) = y_\mu^a + z_\mu^a(\tau)$, where $y_\mu^a = \int\limits_0^1 d\tau x_\mu^a(\tau)$ is the position of the trajectory, whereas the vector $z_\mu^a(\tau)$ describes its shape, both of which should be averaged over. Finally, the magnetic charges of monopoles are described by six root vectors of the group $SU(3)$: $\mathbf{q}_1 = (1/2, \sqrt{3}/2)$, $\mathbf{q}_2 = (-1, 0)$, $\mathbf{q}_3 = (1/2, -\sqrt{3}/2)$, $\mathbf{q}_{-\alpha} = -\mathbf{q}_\alpha$.

The θ-term by which we are going to extend the theory (1) originally having the form $-\frac{i\theta g^2}{32\pi^2}\int d^4x\mathbf{G}_{\mu\nu}\tilde{\mathbf{G}}_{\mu\nu}$ can be rewritten modulo full derivatives as $\frac{i\theta g^2}{8\pi^2}\int d^4x\mathbf{A}_\mu\mathbf{j}_\mu^{\mathrm{mon}}$. Here, $\tilde{\mathcal{O}}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\lambda\rho}\mathcal{O}_{\lambda\rho}$, $\mathbf{G}_{\mu\nu} \equiv \mathbf{F}_{\mu\nu} + \mathbf{F}_{\mu\nu}^{\mathrm{mon}}$, and the monopole field strength tensor, $\mathbf{F}_{\mu\nu}^{\mathrm{mon}}$, is related to the current $\mathbf{j}_\mu^{\mathrm{mon}}$ according to the modified Bianchi identities: $\partial_\mu\tilde{\mathbf{F}}_{\mu\nu}^{\mathrm{mon}} = \mathbf{j}_\nu^{\mathrm{mon}}$.

Let us now concentrate ourselves on the derivation of an effective field theory describing the monopole ensemble. The desired monopole part of the partition function, $\mathcal{Z}_{\mathrm{mon}}\left[\mathbf{A}_\mu\right]$, (which should then be averaged *w.r.t.* the action $\frac{1}{4}\int d^4x\mathbf{F}_{\mu\nu}^2$) is nothing else, but the statistical weight of the N-monopole configuration,

$$\mathcal{Z}\left[\mathbf{j}_\mu^{\mathrm{mon}}, \mathbf{A}_\mu\right] = \exp\left[-\frac{1}{2}\int d^4x\int d^4y\mathbf{j}_\mu^{\mathrm{mon}}(x)D_0(x-y)\mathbf{j}_\mu^{\mathrm{mon}}(y) - \frac{i\theta g^2}{8\pi^2}\int d^4x\mathbf{A}_\mu\mathbf{j}_\mu^{\mathrm{mon}}\right], \tag{3}$$

summed up over the grand canonical ensemble of monopoles:

$$\mathcal{Z}_{\mathrm{mon}}\left[\mathbf{A}_\mu\right] = \sum_{N=1}^{\infty}\frac{\zeta^N}{N!}\left\langle\mathcal{Z}\left[\mathbf{j}_\mu^{\mathrm{mon}}, \mathbf{A}_\mu\right]\right\rangle. \tag{4}$$

Here, $\zeta \propto \mathrm{e}^{-S_{\mathrm{mon}}}$ is the fugacity (Boltzmann factor) of a single monopole loop, which has the dimension (mass)^4. As far as the action of a single a-th loop is concerned, it obeys the estimate $S_{\mathrm{mon}} \propto \frac{1}{g^2}\int\limits_0^1 d\tau\sqrt{\left(\dot{z}_\mu^a\right)^2}$, where the integral expressing the length of the loop is supposed to be of the same order of magnitude for all the loops. Next, the average over monopole loops in eq. (4) has the following form:

$$\left\langle\mathcal{O}\left[\mathbf{j}_\mu^{\mathrm{mon}}\right]\right\rangle = \prod_{a=1}^{N}\int d^4y^a\int\mathcal{D}z^a\mu\left[z^a\right]\sum_{\alpha_a=\pm1,\pm2,\pm3}\mathcal{O}\left[\mathbf{j}_\mu^{\mathrm{mon}}\right] \tag{5}$$

In eq. (5), $\mu\left[z^a\right]$ stands for a certain rotation- and translation invariant measure of integration over the shapes of monopole loops. Similarly to the case of compact QED [10], the concrete form of this measure is irrelevant to the final result for the effective action. It is only important that this measure is normalized in the standard way, $\langle 1\rangle = 1$, and from now on this normalization condition will be implied.

Equation (4) can further be rewritten as follows:

$$\mathcal{Z}_{\rm mon}\left[\mathbf{A}_\mu\right]=\sum_{N=1}^{\infty}\frac{\zeta^N}{N!}\int\mathcal{D}\mathbf{j}_\mu\mathcal{Z}\left[\mathbf{j}_\mu,\mathbf{A}_\mu\right]\left\langle\delta\left(\mathbf{j}_\mu-\mathbf{j}_\mu^{\rm mon}\right)\right\rangle=$$

$$=\int\mathcal{D}\mathbf{j}_\mu\mathcal{Z}\left[\mathbf{j}_\mu,\mathbf{A}_\mu\right]\int\mathcal{D}\mathbf{l}_\mu\exp\left(-i\int d^4x\mathbf{l}_\mu\mathbf{j}_\mu\right)\sum_{N=1}^{\infty}\frac{\zeta^N}{N!}\left\langle\exp\left(i\int d^4x\mathbf{l}_\mu\mathbf{j}_\mu^{\rm mon}\right)\right\rangle. \quad (6)$$

The sum over N here is equal to

$$\exp\left[2\zeta\int d^4y\sum_{\alpha=1}^{3}\int\mathcal{D}z\mu[z]\cos\left(g_m\mathbf{q}_\alpha\oint dz_\mu(\tau)\mathbf{l}_\mu(x(\tau))\right)\right]\simeq$$

$$\simeq\exp\left[2\zeta\int d^4y\sum_{\alpha=1}^{3}\cos\left(\frac{\mathbf{q}_\alpha\left|\mathbf{l}_\mu(y)\right|}{\Lambda}\right)\right]. \quad (7)$$

The derivation of this formula was based on a certain evaluation of the integral over shapes of monopole loops in the strong coupling limit $g\gg 1$ analogous to that of compact QED [10] ([2]). Besides that, it has been assumed that characteristic distances between loops $\sqrt{\left(y_\mu^a\right)^2}$ as well as characteristic loop sizes $\sqrt{\left(z_\mu^a\right)^2}$ are of the same order of magnitude for all a's. The respective UV momentum cutoff Λ (which is much larger than $1/\sqrt{\left(z_\mu^a\right)^2}$) obeys the estimate $\Lambda\sim g\sqrt{\left(y_\mu^a\right)^2}/\left(z_\mu^a\right)^2$. Finally, in eq. (7), $|\mathbf{l}_\mu|$ denotes the modulus of the vector only *w.r.t.* the Lorentz indices, but not *w.r.t.* the Cartan ones, *i.e.* $|\mathbf{l}_\mu|\equiv\left(|\mathbf{l}_\mu|^1,|\mathbf{l}_\mu|^2\right)=\left(\sqrt{l_\mu^1 l_\mu^1},\sqrt{l_\mu^2 l_\mu^2}\right)$.

We should now carry out the integration over the Lagrange multiplier $\mathbf{l}_\mu$ stemming from eqs. (6) and (7). Since the field $\mathbf{l}_\mu$ has no kinetic term, this can be done in the saddle-point approximation. For every Cartan index $a=1,2$, the respective saddle-point equation reads $\frac{l_\mu^a}{|\mathbf{l}_\mu|^a}\sum\limits_{\alpha=1}^{3}q_\alpha^a\sin\left(\frac{\mathbf{q}_\alpha|\mathbf{l}_\mu|}{\Lambda}\right)=-\frac{i\Lambda}{2\zeta}j_\mu^a$. It can be solved *w.r.t.* $\mathbf{q}_\alpha\left|\mathbf{l}_\mu\right|$ by adapting for l_μ^a the following *Ansatz* $l_\mu^a=|\mathbf{l}_\mu|^a j_\mu^a/|\mathbf{j}_\mu|^a$ and representing $|\mathbf{j}_\mu|^a$ as $\sum\limits_{\alpha=1}^{3}q_\alpha^a j^{(\alpha)}$, where $j^{(1)}=\left(|\mathbf{j}_\mu|^1/\sqrt{3}+|\mathbf{j}_\mu|^2\right)/\sqrt{3}$, $j^{(2)}=-2|\mathbf{j}_\mu|^1/3$, $j^{(3)}=\left(|\mathbf{j}_\mu|^1/\sqrt{3}-|\mathbf{j}_\mu|^2\right)/\sqrt{3}$. The so-obtained solution reads $\mathbf{q}_\alpha\left|\mathbf{l}_\mu\right|=-i\Lambda\,\operatorname{arcsinh}\left(\Lambda j^{(\alpha)}/(2\zeta)\right)$. By making use of it and performing the rescaling of fields $g\mathbf{A}_\mu=\mathbf{A}_\mu^{\rm new}$, $g\mathbf{j}_\mu=\mathbf{j}_\mu^{\rm new}$ we eventually get from eqs. (1), (3), and (6) the following expression for the full partition function: $\mathcal{Z}=\int\mathcal{D}\mathbf{A}_\mu\mathcal{D}\mathbf{j}_\mu{\rm e}^{-S}$, where

$$S=\frac{1}{4g^2}\int d^4x\mathbf{F}_{\mu\nu}^2+\frac{1}{2g^2}\int d^4x\int d^4y\mathbf{j}_\mu(x)D_0(x-y)\mathbf{j}_\mu(y)+\frac{i\theta}{8\pi^2}\int d^4x\mathbf{A}_\mu\mathbf{j}_\mu+V\left[\mathbf{j}_\mu\right]. \quad (8)$$

Here,

$$V\left[\mathbf{j}_\mu\right]=\sum_{\alpha=1}^{3}\int d^4x\left[\frac{\Lambda}{g}j^{(\alpha)}\operatorname{arcsinh}\left(\frac{\Lambda}{2g\zeta}j^{(\alpha)}\right)-2\zeta\sqrt{1+\left(\frac{\Lambda}{2g\zeta}j^{(\alpha)}\right)^2}\,\right] \quad (9)$$

([2])Note that similarly to the case of 4D compact QED [7,11], eq. (7) can also be derived by using the lattice regularization.

is the multivalued potential of monopole currents.

As we shall see below, it is convenient to unify the kinetic term of the field $\mathbf{A}_\mu$ and the interaction of monopole currents by introducing the following antisymmetric tensor field (else called Kalb-Ramond field [12]) ([3]) $\mathbf{B}_{\mu\nu} = \mathbf{F}_{\mu\nu} + \mathbf{h}_{\mu\nu}$. Here, the field $\mathbf{h}_{\mu\nu}$ is unambiguously related to the current $\mathbf{j}_\mu$ as $\mathbf{h}_{\mu\nu} = -\varepsilon_{\mu\nu\lambda\rho}\partial^x_\lambda \int d^4y D_0(x-y)\mathbf{j}_\rho(y)$ and thus obeys the modified Bianchi identities $\partial_\mu \tilde{\mathbf{h}}_{\mu\nu} = \mathbf{j}_\nu$. ([4]) In terms of the field $\mathbf{B}_{\mu\nu}$, the action (8) takes the following more compact form:

$$S = \frac{1}{4g^2}\int d^4x \mathbf{B}^2_{\mu\nu} - \frac{i\theta}{32\pi^2}\int d^4x \mathbf{B}_{\mu\nu}\tilde{\mathbf{B}}_{\mu\nu} + V\left[\partial_\mu \tilde{\mathbf{B}}_{\mu\nu}\right]. \tag{10}$$

In what follows, we shall be interested in the string representation of the Wilson loop defined as $\langle W(C)\rangle = \frac{1}{3}\left\langle \mathrm{tr}\, P\, \exp\left(i\oint\limits_C dx_\mu \mathbf{A}^{\mathrm{tot}}_\mu \mathbf{T}\right)\right\rangle$. Here, $\mathbf{A}^{\mathrm{tot}}_\mu$ is the total vector potential which includes also the monopole contributions, and $\mathbf{T} = \left(\frac{\lambda_3}{2}, \frac{\lambda_8}{2}\right)$ with $\lambda_{3,8}$ denoting the Gell-Mann matrices. Analogously to the case when monopoles are absent, Stokes theorem yields for the Wilson loop the following expression:

$$\langle W(C)\rangle = \frac{1}{3}\left\langle \mathrm{tr}\, \exp\left[\frac{i}{2}\int d^4x \mathbf{G}_{\mu\nu}\mathbf{T}\Sigma_{\mu\nu}\right]\right\rangle \tag{11}$$

Here, $\Sigma_{\mu\nu}[x,\Sigma] = \int\limits_{\Sigma(C)} d\sigma_{\mu\nu}(x(\xi))\delta(x-x(\xi))$ is the vorticity tensor current defined at a certain surface $\Sigma(C)$ bounded by the contour C and parametrized by the vector $x_\mu(\xi)$ with $\xi = (\xi^1, \xi^2)$ standing for the 2D coordinate. In a derivation of eq. (11), we have omitted the path ordering, which is possible due to the fact that both λ_3 and λ_8 are diagonal. Note that by virtue of the quantization condition $gg_m = 4\pi$ one can prove that eq. (11) is indeed independent of the form of the surface Σ.

The average in eq. (11) is first performed over the free part of the $\mathbf{A}^{\mathrm{tot}}_\mu$-action, $\frac{1}{4g^2}\int d^4x \mathbf{F}^2_{\mu\nu}$, after which the result should be weighted with $\mathcal{Z}\left[\frac{1}{g}\mathbf{j}^{\mathrm{mon}}_\mu, \frac{1}{g}\mathbf{A}_\mu\right]$ and summed up over the grand canonical ensemble of monopoles in the sense of eqs. (4)-(5). This procedure can be simplified by rewriting the Wilson loop in terms of the above-introduced field $\mathbf{B}_{\mu\nu}$ as follows:

$$\langle W(C)\rangle = \frac{1}{3}\sum_{\alpha=1}^{3}\left\langle \exp\left(\frac{i}{2}\int d^4x \mathbf{B}_{\mu\nu}\mathbf{Q}_\alpha \Sigma_{\mu\nu}\right)\right\rangle. \tag{12}$$

Here, $\langle\ldots\rangle$ stands for the average *w.r.t.* the action (10), and the vectors $\mathbf{Q}_\alpha$'s, which denote the charges of a quark of the α's colour *w.r.t.* the diagonal gluons, have the following form: $\mathbf{Q}_1 = \left(-\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$, $\mathbf{Q}_2 = \left(\frac{1}{2}, \frac{1}{2\sqrt{3}}\right)$, $\mathbf{Q}_3 = \left(0, -\frac{1}{\sqrt{3}}\right)$. Besides that, in the derivation of eq. (12) we have used the identity $\mathrm{tr}\,\exp(i\mathbf{a}\mathbf{T}) = \sum\limits_{\alpha=1}^{3}\exp(i\mathbf{a}\mathbf{Q}_\alpha)$ valid for an arbitrary vector $\mathbf{a}$.

In the low-energy limit, $\left|j^{(\alpha)}\right| \ll g\zeta/\Lambda$, the monopole potential (9) becomes a quadratic functional. This yields the kinetic term of the field $\mathbf{B}_{\mu\nu}$, $V\left[\partial_\mu \tilde{\mathbf{B}}_{\mu\nu}\right] \to \frac{1}{12\eta^2}\int d^4x \mathbf{H}^2_{\mu\nu\lambda}$, where $\eta \equiv g\sqrt{3\zeta}/\Lambda$ and $\mathbf{H}_{\mu\nu\lambda} = \partial_\mu \mathbf{B}_{\nu\lambda} + \partial_\lambda \mathbf{B}_{\mu\nu} + \partial_\nu \mathbf{B}_{\lambda\mu}$. The Wilson loop (12) in such a low-energy limit takes the form

([3])In the formal language, this equation represents the Hodge decomposition theorem.

([4]) Clearly, the same Legendre transformation, which made from the current $\mathbf{j}^{\mathrm{mon}}_\mu$ the dynamical field $\mathbf{j}_\mu$, makes from $\mathbf{F}^{\mathrm{mon}}_{\mu\nu}$ the dynamical field $\mathbf{h}_{\mu\nu}$.

$$\langle W(C)\rangle|_{\text{low en.}} = \frac{1}{\mathcal{Z}_{\text{low en.}}}\times$$

$$\times \frac{1}{3}\sum_{\alpha=1}^{3}\int \mathcal{D}\mathbf{B}_{\mu\nu}\exp\left\{-\int d^4x\left[\frac{1}{12\eta^2}\mathbf{H}^2_{\mu\nu\lambda}+\frac{1}{4g^2}\mathbf{B}^2_{\mu\nu}-\frac{i\theta}{32\pi^2}\mathbf{B}_{\mu\nu}\tilde{\mathbf{B}}_{\mu\nu}-\frac{i}{2}\mathbf{B}_{\mu\nu}\mathbf{Q}_\alpha\Sigma_{\mu\nu}\right]\right\}, \tag{13}$$

where $\mathcal{Z}_{\text{low en.}}$ is given by the second line of eq. (13) with $\Sigma_{\mu\nu}$ set to zero. Carrying out the $\mathbf{B}_{\mu\nu}$-integration, whose details are outlined in the Appendix A, we get the following result:

$$\langle W(C)\rangle|_{\text{low en.}} = \exp\left\{-\frac{1}{12}\left[2g^2\oint\limits_C dx_\mu\oint\limits_C dy_\mu D_m(x-y)+\right.\right.$$

$$\left.\left.+\eta^2\int d^4x\int d^4y D_m(x-y)\left(\Sigma_{\mu\nu}(x)\Sigma_{\mu\nu}(y)+\frac{i\theta g^2}{8\pi^2}\Sigma_{\mu\nu}(x)\tilde{\Sigma}_{\mu\nu}(y)\right)\right]\right\}. \tag{14}$$

Here,

$$m=\frac{\eta}{g}\sqrt{1+\left(\frac{\theta g^2}{8\pi^2}\right)^2} \tag{15}$$

is the mass of the field $\mathbf{B}_{\mu\nu}$, and $D_m(x)=mK_1(m|x|)/\left(4\pi^2|x|\right)$ is the massive boson propagator with K_1 standing for the modified Bessel function.

Upon the derivative expansion of eq. (14) (analogous to the expansion performed in ref. [13] within the stochastic vacuum model of QCD), one gets as a few first local string terms the usual Nambu-Goto and rigidity terms, responsible for confinement of electric charges and stability of strings. Besides that, this expansion yields string θ-term [14] ([5]) equal to $ic\nu$, where $\nu\equiv(2\pi)^{-1}\int d^2\xi\sqrt{\hat{g}}\hat{g}^{ab}\left(\partial_a t_{\mu\nu}\right)\left(\partial_b\tilde{t}_{\mu\nu}\right)$ is the number of self-intersections of the world sheet Σ. Here we have adapted the standard notations: $\hat{g}^{ab}=(\partial^a x_\mu(\xi))(\partial^b x_\mu(\xi))$ denotes the induced metric tensor of the surface, $\hat{g}=\det\left|\left|\hat{g}^{ab}\right|\right|$, and $t_{\mu\nu}=\varepsilon^{ab}\left(\partial_a x_\mu(\xi)\right)\left(\partial_b x_\nu(\xi)\right)/\sqrt{\hat{g}}$ stands for its extrinsic curvature tensor. The coupling constant c can be calculated analogously to refs. [13, 15] and reads $c=-\frac{\theta}{768}\left(\frac{g\eta}{\pi m}\right)^2$. Since the obtained string θ-term is proportional to the number of self-intersections of the world sheet, it might be relevant to the solution of the problem of crumpling of the world sheet, as it has been first mentioned in ref. [14]. In particular, in the extreme strong coupling limit $g\to\infty$, $c\to-\frac{\pi^2}{12\theta}$, and therefore self-intersections are weighted in the partition function with the desired factor $(-1)^\nu$ at $\theta=\pi/12$.

Notice also that owing to the field-theoretical θ-term, in this limit the dual mass (15) is very large:

$$m\to\frac{\theta g\eta}{8\pi^2}. \tag{16}$$

This situation is opposite to the case without θ-term, where the dual mass equal to η/g vanishes in the extreme strong coupling limit, since $\eta(g)\propto \mathrm{e}^{-\text{const}/g^2}\to 1$. Therefore, contrary to that case, at $\theta\neq 0$, the expansion of the Σ-dependent part of the action (14) in powers of the derivatives *w.r.t.* ξ^a's [13, 15], being nothing else but the expansion in the inverse powers

([5]) The string θ-term can also be derived from the instanton gas model of QCD, as it has been done in ref. [15].

of m, converges fastly. In particular, this means that all the terms of this expansion higher in the derivatives than the Nambu-Goto, rigidity, and θ-ones are irrelevant, since they are suppressed by the higher powers of the mass.

Clearly, all the lowest local string terms, *i.e.* the Nambu-Goto and rigidity ones, as well as the above-discussed θ-term, depend explicitly on the form of the world sheet Σ. This is the consequence of the fact that in a derivation of eq. (13) we have taken into account only one (namely, real) branch of the monopole potential (9). However, as it has been argued first for the case of compact QED in ref. [6] and then discussed for the 3D $SU(3)$-inspired case in ref. [5], the world-sheet independence of the Wilson loop becomes restored upon the summation over all the complex-valued branches of the potential (9) at every space-time point.

Notice that although the low-energy limits of confining string theories obtained in compact QED and in the $SU(3)$-inspired case under study have the same functional form, the full expressions are different since in the $SU(3)$-case the $\mathbf{B}_{\mu\nu}$-field has two components. Moreover, the obtained results can be generalized to the $SU(N)$-inspired case along with the lines of ref. [9]. There the fields $\mathbf{B}_{\mu\nu}$ and $\mathbf{j}_\mu$ have $(N-1)$ components, but the functional form of the action (8)-(9) remains the same modulo the fact that the fields $j^{(\alpha)}$'s are given by more complicated combinations of the fields $|\mathbf{j}_\mu|^a$'s. Let us finally discuss the asymptotic behaviour $m(N)$ at $N \gg 1$. Note first of all that the strong coupling limit under consideration is still accessible at large N despite the fact that $g \sim 1/\sqrt{N}$. That is because the strong coupling limit implies only that g should be larger than some critical value, which itself scales as $1/\sqrt{N}$. Taking into account that $\eta(g) \propto \mathrm{e}^{-\mathrm{const}/g^2}$, we see that according to eq. (15), the dual mass behaves in the large-N limit as $\sqrt{N}\mathrm{e}^{-\mathrm{const}\,N}$. Besides that, in the $SU(N)$-case there exist $N(N-1)/2$ vectors $\mathbf{q}_\alpha$'s with $\alpha > 0$ and consequently the same amount of mass terms of $(N-1)$ fields, which emerge from the expansion of cosines in the respective sine-Gordon theory describing these fields. Therefore, the number of mass terms of every field is of the order of N, *i.e.* each mass scales as $\sqrt{N}$ owing to this fact. These two observations considered together lead to the conclusion that at $N \gg 1$, $m \sim N\mathrm{e}^{-\mathrm{const}\,N}$.

However, in the large-N limit, the model under study becomes less and less relevant to QCD. That is because the number of off-diagonal fields disregarded within the Abelian dominance hypothesis, equal to $\left(N^2 - N\right)$, significantly exceeds in this limit the number of diagonal fields kept, equal to $(N-1)$. The problem of accounting for off-diagonal degrees of freedom deserves special investigations and will be considered in future publications.

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Appendix A. Details of integration over the $\mathbf{B}_{\mu\nu}$*-field in eq. (13).* – The desired integration is Gaussian and can obviously be performed by finding the inverse to the quadratic part of the $\mathbf{B}_{\mu\nu}$-field action. Since the integral under study factorizes into the product of the integrals over $B^a_{\mu\nu}$'s, we should only solve the following equation in the momentum representation:

$$\frac{1}{2}\left(\frac{p^2}{\eta^2}\hat{P} + \frac{1}{g^2}\hat{1} - \frac{i\theta}{16\pi^2}\varepsilon\right)_{\mu\nu\alpha\beta} G_{\alpha\beta\lambda\rho}(p) = \hat{1}_{\mu\nu\lambda\rho}. \qquad (A.1)$$

Here, the following projection operators were introduced (see *e.g.* [10]):

$$\hat{P}_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\mathcal{P}_{\mu\alpha}\mathcal{P}_{\nu\beta} - \mathcal{P}_{\mu\beta}\mathcal{P}_{\nu\alpha}\right), \ \hat{1}_{\mu\nu\alpha\beta} = \frac{1}{2}\left(\delta_{\mu\alpha}\delta_{\nu\beta} - \delta_{\mu\beta}\delta_{\nu\alpha}\right),$$

where $\mathcal{P}_{\mu\nu} = \delta_{\mu\nu} - p_\mu p_\nu / p^2$. In what follows, we shall for brevity omit the indices and use the notation $(\mathcal{O}\cdot\mathcal{O}')_{\mu\nu\lambda\rho} \equiv \mathcal{O}_{\mu\nu\alpha\beta}\mathcal{O}'_{\alpha\beta\lambda\rho}$. Then, representing G as $G^{(1)} + iG^{(2)}$, we get the following equations for the real and imaginary parts of eq. (A.1):

$$\left(\frac{p^2}{\eta^2}\hat{P} + \frac{1}{g^2}\hat{1}\right)\cdot G^{(1)} + \frac{\theta}{16\pi^2}\varepsilon\cdot G^{(2)} = 2\hat{1}, \ \left(\frac{p^2}{\eta^2}\hat{P} + \frac{1}{g^2}\hat{1}\right)\cdot G^{(2)} - \frac{\theta}{16\pi^2}\varepsilon\cdot G^{(1)} = 0.$$

Applying further to the second of these equations the operation $\varepsilon\cdot$, it is possible to resolve it *w.r.t.* $G^{(1)}$. Substituting the so-obtained expression into the first equation, we get the following equation for $G^{(2)}$:

$$\left(\frac{p^2}{\eta^2} + \frac{1}{g^2}\right)\left\{\frac{p^2}{\eta^2}\left[\left(\hat{P} - \hat{1}\right)\cdot\varepsilon + \varepsilon\cdot\left(\hat{P} - \hat{1}\right)\right] + \left(\frac{p^2}{\eta^2} + \frac{1}{g^2}\right)\varepsilon\right\}\cdot G^{(2)} + \frac{\theta^2}{64\pi^4}\varepsilon\cdot G^{(2)} = \frac{\theta}{2\pi^2}\hat{1}.$$

Finally, seeking for $G^{(2)}$ in the form of the *Ansatz* $G^{(2)} = f(p)\varepsilon$ and using the equation $\left[\left(\hat{P} - \hat{1}\right)\cdot\varepsilon + \varepsilon\cdot\left(\hat{P} - \hat{1}\right)\right]\cdot\varepsilon = -4\hat{1}$, we find

$$f = \frac{\theta g^2\eta^2}{8\pi^2\left(p^2 + m^2\right)}, \ G^{(1)} = \frac{2(g\eta)^2}{p^2 + m^2}\left[\frac{p^2}{\eta^2}\left(\hat{1} - \hat{P}\right) + \frac{1}{g^2}\hat{1}\right],$$

where the mass m of the field $\mathbf{B}_{\mu\nu}$ is given by eq. (15). One can now straightforwardly obtain eq. (14) of the main text by passing back to the coordinate representation, using Stokes theorem, and the fact that for every α, $\mathbf{Q}_\alpha^2 = 1/3$.

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